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Temporal sampling of backscattered sonar signals

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This article considers the subject of the temporal sampling of backscattered sound from an active sonar system. Here, it is demonstrated how the beam patterns of the sonar, when considered along with the velocity of the animal, prescribe a minimal sampling rate such that an unaliased version of the backscattered signal can be constructed. Thus, an equation is derived which relates a minimum sampling interval to the maximal angular velocity of targets which are moving through the beams. Observation of this minimal temporal sampling criteria is necessary in order to obtain unaliased data.

Key words: active sonar systems, sampling theory, temporal sampling.

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Introduction

Distributions of animals occur naturally in space and vary in time. In this context, the ultimate goal of the type of instrumentation system considered in this article is to obtain realistic information about the parameters which govern both the state and the evolution of marine ecosystems. Sonars have many advantages for providing non-invasive, rapid, and quantitative information about the statics and dynamics of these ecosystems which are very difficult to obtain with other types of instruments. Signal-processing methodologies describe in a mathematical way the relationships between signals, i.e. images or one-dimensional parameters, and the physical parameters which we, as observational scientists, seek to measure. For example, sampling theorems govern relationships between the temporal and spatial sampling intervals of data which are necessary to portray their unaliased representation (Bracewell, 1978; Oppenheim and Schaefer, 1989).

An interesting topic concerns the temporal evolution of the field of view of the backscatter target strength by a sonar system, a function of time so that the field is $B_s(\mathbf{r},t)$. The view taken here is that the system is imaging a number of animals, whose dimensions are small with respect to a beam width. This allows the target to be modeled as point targets. A very interesting question that arises in understanding the parameters over which such a system operates is related to the sampling frequency ΔT. That is, how often does such a system need to sample a set of targets moving with velocities $\mathbf{v}(\mathbf{r})$ so that the system does not alias the recorded information? The question is answered by proving a relationship between a scaled version of the self-convolution of the transducer aperture and the velocity at which targets are moving.

Temporal sampling of time varying signals by sonars

Theory

As is widely appreciated, pelagic ecosystems vary both in space and in time. Since the purpose of any instrumentation system is to obtain a ‘‘snap shot’’ of these processes, it is important to understand how these varying processes affect an estimate of the time-varying backscatter function. This understanding can be used for both instrument design and data interpretation.

Under the assumption that the around trip propagation time for sound is short with respect to animal movement and also the interframe interval of a system, the assumption can be made that the image of the backscattered distribution of target strengths is virtually instantaneous $B_s(\mathbf{r},t_0)$. The question then arises as to how often it is necessary to sample this changing field so as to avoid temporal aliasing of the data.

This problem has been approached by looking at the range of animal velocities that can be observed with different interframe sampling intervals. This is because our particular interests are in tracking individuals. Intuitively, the faster an animal is swimming, the more rapidly the trajectory will need to be sampled. On the

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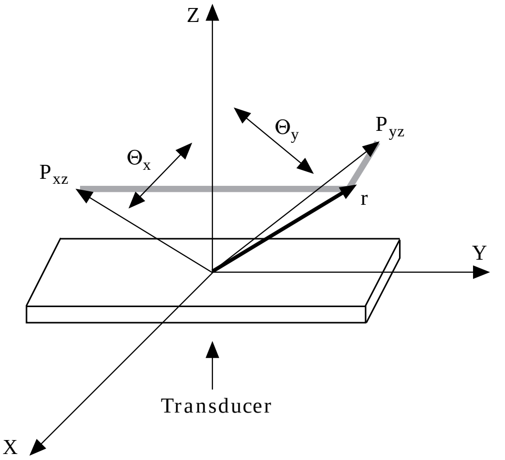

Figure 1. The coordinate system for the transducer (aligned with the XY plane), showing the projection of an arbitrary vector **r** onto the XZ (P_{xz}) and YZ (P_{yz}) planes. Θ_x and Θ_y are the pointing angles of the vector **r**, relative to broadside (the z axis).

other hand, if the beams of a sonar system are very wide, the intensity of the reflected sound will change little as a function of animals' position. It is thus reasonable to expect that the sampling rate will increase as animal velocity increases and will decrease as the system beam width decreases. The problem can be solved by transforming the spatial patterns of the imaging system into the time domain by writing down an equation for the trajectory of a target as a function of time. Straightforward Fourier transformation of this function followed by inspection of the result permits an estimate of the minimal sampling interval.

Consider first the relationship between a rectangular transducer of dimensions L by K and its beam pattern, represented by

$$BP^2(s_x,s_y)=\frac{\sin^2(\pi s_x L/\lambda)\sin^2(\pi s_y K/\lambda)}{(\pi s_x L/\lambda)^2(\pi s_y K/\lambda)^2}. \qquad (1)$$

Here, $s_x=\sin(\theta_x)$ (angular displacement in radians relative to the x axis), $s_y=\sin(\theta_y)$ (angular displacement in radians relative to the y axis), and λ is the wavelength. Note that K/λ is the length of the transducer in wavelengths, and L/λ is the width in wavelengths. The coordinate system for this equation is illustrated in Figure 1.

Assuming that the same transducer is used for transmitting and receiving, the complete equation for the intensity of the backscattered sound for a point target of target strength $B_s(\theta_x,\theta_y,d)$ located at position (θ_x,θ_y,d), where d is the range from the transducer to the target, can be obtained by convolving the beam patterns with a delta function at this location and multiplying by the intensity of the reflector. This yields

$$I(\theta_x,\theta_y,d)=\frac{e^{-2\alpha d}}{d^4}BP^4(\theta_x,\theta_y)$$

$$=\frac{e^{-2\alpha d}}{d^4}B_s(\theta_x,\theta_y,d)\frac{\sin^4(\pi\theta_x L/\lambda)\sin^4(\pi\theta_y K/\lambda)}{(\pi\theta_x L/\lambda)^4(\pi\theta_y K/\lambda)^4} \qquad (2)$$

Here, α is the attenuation coefficient, and a source of unit intensity is assumed. Also assumed is that the field of view of each transducer in the sonar system is small, so that $s_x=\sin(\theta_x)\simeq\theta_x$. Similar arguments apply for y.

Now, the transformation from space to time can be made by assuming that a target is moving with positional coordinate $\mathbf{r}=\mathbf{r}_0+\mathbf{v}t$. Here, r_0 is the position of the target at time $t=0$, and $\mathbf{v}$ is its velocity. Assuming the velocity can be represented as an angular velocity in θ_x and θ_y yields

$$\mathbf{r}(t)=(\theta_x(t),\theta_y(t),z(t))=\left(\theta_{0x}+\Omega_x t,\theta_{0y}+\Omega_y t,z_0+\frac{dz}{dt}t\right).$$

Here, $\Omega_x=2\pi d\theta_x/dt$ and $\Omega_y=2\pi d\theta_y/dt$. Assuming that $\mathbf{r}=0$ at time t_0 (the initial position should not influence the temporal sampling of the system), and that the target is only changing position in angle, and making the substitutions for θ_x and θ_y, we obtain

$$I(t)=\frac{e^{-2\alpha d}}{d^4}BP^4(\Omega_x,\Omega_y)$$

$$=\frac{e^{-2\alpha d}}{d^4}B_s(\theta_x,\theta_y,d)\frac{\sin^4(\pi\Omega_x tL/\lambda)\sin^4(\pi\Omega_y K/\lambda)}{(\pi\Omega_x tL/\lambda)^4(\pi\Omega_y tK/\lambda)^4} \qquad (3)$$

In order to obtain the sampling theorem for the system, it is necessary to take the temporal Fourier transform of Equation (3). Although this looks somewhat ominous, the expression can be easily obtained in a form that is suitable for this analysis by using the well-known convolution theorem in conjunction with the Fourier transform of the function $\mathrm{sinc}^2(at)=\sin^2(\pi at)/(\pi at)^2$:

$$\mathscr{F}\{\mathrm{sinc}^2(at)\}=\frac{1}{|a|}\Lambda\left(\frac{f}{a}\right) \qquad (4)$$

$$\Lambda\left(\frac{f}{a}\right)=1-\left|\frac{f}{a}\right| \text{ for } \left|\frac{f}{a}\right|\leq 1 \qquad (5)$$

$$=0 \text{ otherwise.} \qquad (6)$$

Here, f stands for frequency. Applying the multiple product theorem for convolution (Bracewell, 1978):

$$F\{g_1(t)g_2(t)g_3(t)f_4(t)\}=G_1(f)*G_2(f)*G_3(f)*G_4(f) \qquad (7)$$

yields

$$\mathscr{F}\{I(t)\}$$

$$=\mathscr{K}\Lambda\left(\frac{\lambda f}{\Omega_x L}\right)*\Lambda\left(\frac{\lambda f}{\Omega_x L}\right)*\Lambda\left(\frac{\lambda f}{\Omega_y K}\right)*\Lambda\left(\frac{\lambda f}{\Omega_y K}\right) \qquad (8)$$

where the symbol * denotes convolution and the K denotes a set of scaling coefficients which do not change as the target is considered to be at constant range.

Simply stated, the Fourier transform of expression (3) is the autoconvolution (in Fourier space) of two sets of two triangle functions whose bases are $2\Omega_x L/\lambda$ and $2\Omega_y K/\lambda$ wide. Although it is possible to write this function explicitly, the important aspect of it for the purposes of this section is its width. The width of this function is simply the sum of the widths of the individual triangle functions. This follows from the fact that the width of the convolution of two triangle functions is the sum of the width of the individual functions. One attractive aspect of this function is that it is of limited extent in Fourier Space. Sampling the time-varying function on a grid of delta functions ΔT is equivalent to convolving the spectrum with a grid of delta functions of inverse temporal duration $1/\Delta T$. Thus, it is possible to obtain an unaliased spectrum of a target traversing the sonar beam at a fixed range d by sampling at intervals ΔT, if and only if:

$$\Delta T \leq \frac{\lambda}{4(\Omega_x L + \Omega_y K)} \tag{9}$$

If the target is altering its range and direction, which is usually the case, the target position must be written out as a function of this three-dimensional change in location using the same analysis. The extension of this analysis to include the range direction is straightforward and will not be considered further here. In cases where an analytic expression for the Fourier transform may not exist, it may be necessary to compute the transformation numerically.

Example of an application: FishTV

As an example of the application of this theorem, we consider the FishTV system as described in (Jaffe *et al.*, 1995). The system consists of two sets of eight rectangular apertures which are pointed in different directions. The product of the transmitters and receivers is the number of resolvable directions (64) as the system transmits on each beam one at a time and receives in parallel on all eight receiving transducers to obtain the 64 directions. Considering an individual set of look directions, the system impulse response can be computed by measuring the reflection of a delta function as above. Since the apertures are identical transducers which are perpendicular to each other, the system response can be written, including the time varying part of the problem as

$$I(t) = \frac{e^{-2\alpha d}}{d^4} B_s(\theta_x, \theta_y, d)\,\mathrm{sinc}^2(\pi\Omega_x tK/\lambda)\,\mathrm{sinc}^2(\pi\Omega_y tK/\lambda)$$

$$\mathrm{sinc}^2(\pi\Omega_y tL/\lambda)\,\mathrm{sinc}^2(\pi\Omega_x tL/\lambda). \tag{10}$$

Manipulation of these functions, as illustrated above, yields a sampling theorem which says,

$$\Delta T \leq \frac{\lambda}{2(K+L)(\Omega_x + \Omega_y)}. \tag{11}$$

Discussion

In this article, a theorem has been derived which relates a minimum sampling interval to beam patterns for recording unaliased information about the velocity of an animal. Note that this theorem does not guarantee good resolution in computing the velocity. In general, what is recorded is a low pass version of the signal. Higher resolution would require narrower beam patterns, but also more frequent temporal sampling. In some circumstances these sampling theorems may require higher sampling rates than are possible in a sonar survey because of the limited speed of sound in water. In these cases techniques for placing more than one sound in the water via the use of different coded signals may be desirable (Jaffe and Casereau, 1988; Jaffe *et al.*, 1990). Nevertheless, it is comforting to know that for animal velocities below a certain maximum, a given system can unambiguously sample the reflected energy. On the other hand, if animal densities are sparse enough, it might be possible to track the animals even though the stated sampling criteria are not observed.

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